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Distribution of epicenters in the Olami-Feder-Christensen model

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We show that the well established Olami-Feder-Christensen (OFC) model for the dynamics of earthquakes is able to reproduce a striking property of real earthquake data. Recently, it has been pointed out by Abe and Suzuki that the epicenters of earthquakes could be connected in order to generate a graph, with properties of a scale-free network of the Barabási-Albert type. However, only the nonconservative version of the Olami-Feder-Christensen model is able to reproduce this behavior. The conservative version, instead, behaves like a random graph. Besides indicating the robustness of the model to describe earthquake dynamics, those findings reinforce that conservative and nonconservative versions of the OFC model are qualitatively different. Also, we propose a completely different dynamical mechanism that, even without an explicit rule of preferential attachment, generates a scale-free network. The preferential attachment is in this case a ''byproduct'' of the long term correlations associated with the self-organized critical state.

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The concept of self-organized criticality (SOC) was introduced by Bak, Tang, and Wiesenfeld [1] as a possible explanation of scale invariance in nature. Since this seminal work, a great number of cellular automata and coupled map models have been investigated in an attempt to elucidate the essential mechanisms hidden in a wide class of different nonlinear phenomena whose statistics of events (or avalanches) are governed by power laws. However, up to now, although there are successful analytical investigations of several models, one still lacks a general theoretical framework for self-organized criticality. For many models, most of the results are still pure numerical. For a review see, for instance, recent works of Jensen and Turcotte [2,3].

In this context, a model that has been widely studied in the literature is the Olami-Feder-Christensen (OFC) model for the dynamics of earthquakes. The original OFC model, introduced in 1992 [4], is a two-dimensional coupled map model defined on a square lattice, whose dynamical rules were inspired in a spring-block model proposed to describe the dynamics of earthquakes [5]. Earthquakes, in the real world, are associated with many power laws, the best known of them being the Gutenberg-Ritcher law for the distribution of avalanche energies. The OFC model assigns—to each site of a square lattice—a real variable $z_{i,j}$ (energy or tension), initially chosen at random in the interval $[0,z_c)$, where z_c is a threshold value; $z_{i,j}$ increases slowly throughout the lattice and each time that, for a given site, $z_{i,j}$ exceeds z_c, the system relaxes. A fraction $\alpha z_{i,j}$ of the tension of site (i,j) is then distributed to each of its nearest neighbors. As a consequence, the tension of some of its neighbors may also exceed z_c, generating an ''avalanche'' that will only stop when $z_{i,j} < z_c$ again for all sites of the lattice. We have assumed, as usual, open boundary conditions in our simulations.

Within the OFC model, there is a dissipation parameter α. If $\alpha = 0.25$, the total tension in the lattice, $\Sigma z_{i,j}$, is conserved during the avalanching process, in the bulk of the lattice (there is always dissipation in the boundaries). But if $\alpha < 0.25$, there is some dissipation also in the bulk of the system. This model has been widely studied in literature. At the same time, it is a prototype of self-organization in systems with nonconservative relaxation rules (the existence of SOC in the nonconservative models is, up to now, not well understood [6–9]) and a paradigm of the success of SOC ideas, since it is able to reproduce important aspects of the dynamics of earthquakes.

Recently, Abe and Suzuki [10] observed a new power law in the statistics of earthquakes. They analyzed earthquake data from both the district of southern California and Japan, connecting their epicenters in order to generate a graph. Each area analyzed was divided into small cubic cells; they associated with each of these cells a node every time an earthquake started inside it. The epicenters of two successive earthquakes were linked, defining an edge. In this way, the data have been mapped into a complex growing graph that behaves like a scale-free network of the Barabási-Albert type [11]. The degree distribution of the graph decays as a power law. The clustering coefficient and the diameter of a cluster were also calculated, showing small-world network properties [12]. These features have revealed another aspect of earthquakes as a complex critical phenomenon.

We then decided to check whether the Olami-Feder-Christensen model could also predict this striking behavior. We found that the nonconserving version of the model reproduces the behavior of experimental data, even for a very small degree of nonconservation. The degree distribution of the evolving network formed by its epicenters is scale-free. However, the conservative version of the model has a qualitatively different behavior, more similar to a random graph, whose degree distribution is Poisson, indicating that most of the nodes have the same degree and, although random, the corresponding network is much more homogeneous. These results are in agreement with some recent observations that support the claims that conservative and nonconservative versions of the OFC model are quite different. Hergarten and Neugebauer [13] studying the efficiency of the OFC model to predict foreshocks and aftershocks, de Carvalho and Prado

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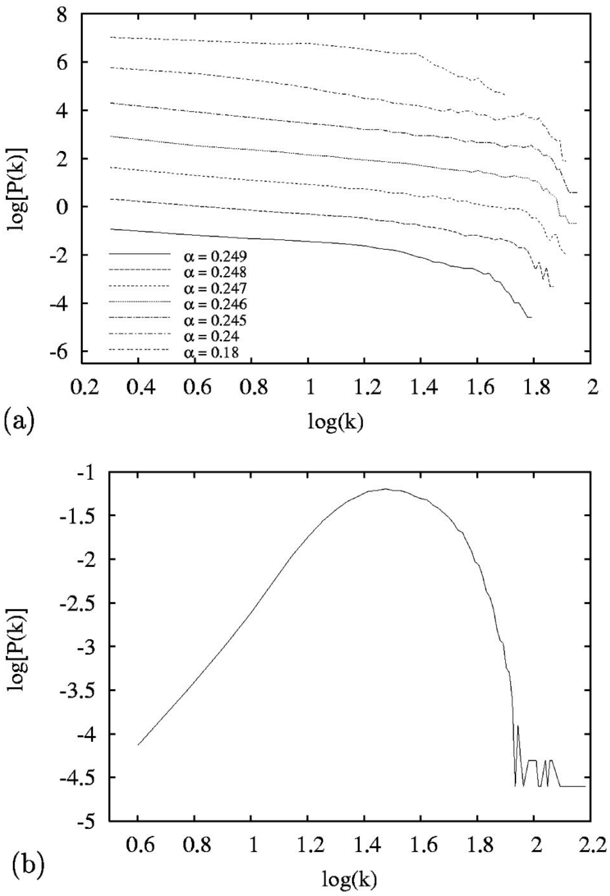

FIG. 1. Degree distribution $P(k)$ for different values of α. (a) Nonconservative regime: the results show a scale-free network behavior in all cases. The curves for $\alpha<0.249$ have been shifted upwards along the y axis for clarity, otherwise they would all coincide. In all cases, $L=200$ and the number of registered epicenters is 10^5. (b) Conservative regime: the degree distribution is similar to a random graph. In this case we have $L=200$ and 10^6 events. Lowering the statistics does not change this behavior. All log scales are base 10.

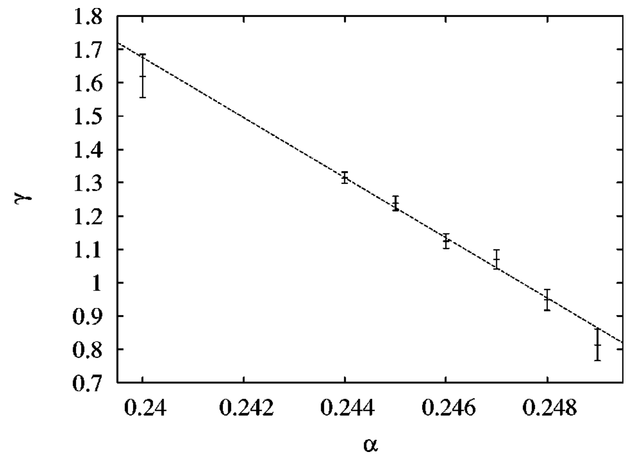

FIG. 2. Exponent γ that characterizes the power-law behavior of $P(k)$. The slope γ seems to increase linearly with α, at least for values of α close to the conservative regime. In all cases, $L=200$ and the number of epicenters is 10^5.

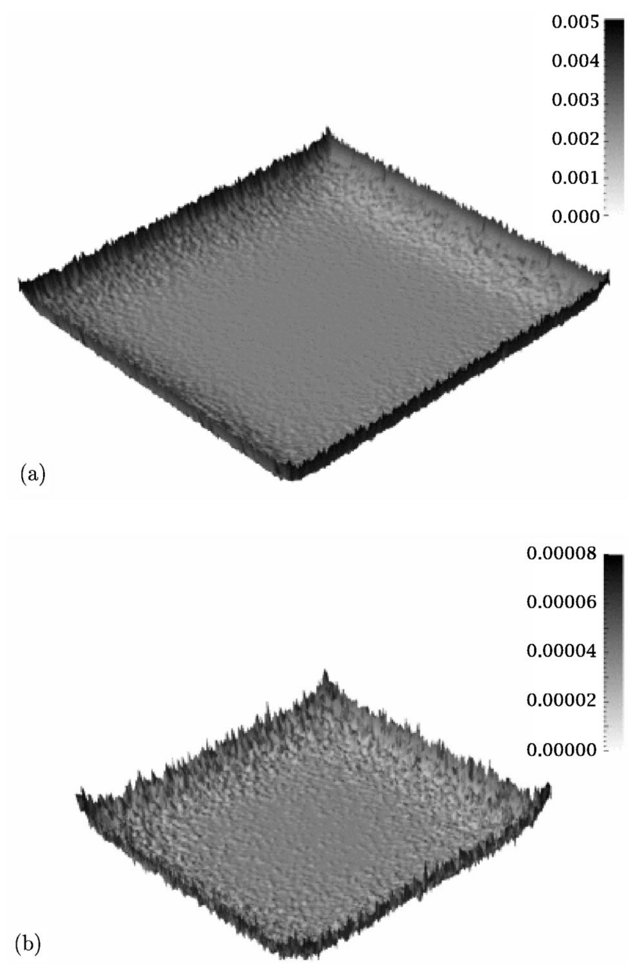

FIG. 3. Spatial distribution of node degrees in the nonconservative case, for $\alpha=0.249$, $L=200$, and 10^5 events. Sites associated with nodes of higher degree are darker and, as one can see, are closer to the boundaries. Panel (b) is a blow up of (a). The 20 sites closer to the boundaries do not appear in the picture, and the scale has been changed in order to show the details of the bulk of the lattice. We can see that the structure of the network observed in (a) is reproduced (and it is not an effect due to the boundaries).

[14], studying the transient behavior of the OFC model, and Miller and Boulter [15], studying the distribution of values at which supercritical sites topple, have also reported qualitatively different behaviors for conservative and nonconservative OFC models.

In a complex graph, the edges are not distributed in a regular way, and not all nodes have the same number of edges. One possible way to characterize complex networks is through its distribution function $P(k)$, which gives the probability that a random selected node has exactly k edges (k is called the *degree* of the node). In a random graph, since the edges are placed randomly among the nodes, the majority of nodes have approximately the same degree, close to the average connectivity $\langle k\rangle$, and the distribution $P(k)$ is a Poisson distribution with a peak at $P(\langle k\rangle)$. Most complex networks, however, have a distribution function $P(k)$ that deviates significantly from a Poisson distribution. In particular, for a large number of networks, associated with a wide class of systems, ranging from the World Wide Web to meta-

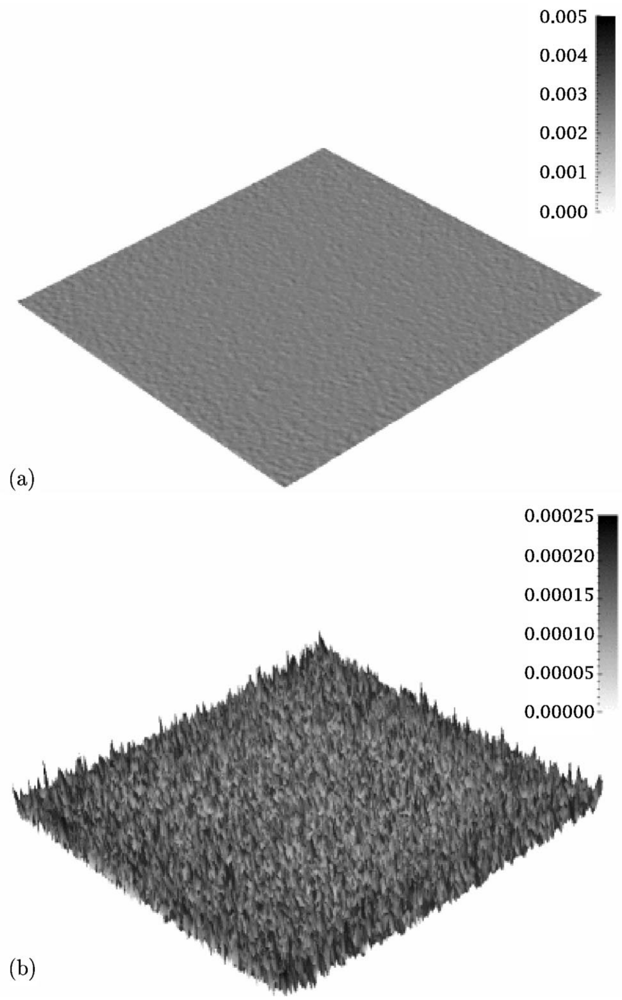

FIG. 4. Spatial distribution of node degrees for the conservative case. Sites associated with nodes of higher degree are darker, $L=200$, and the number of epicenters is 10^6. (a) The same scale of Fig. 3(a) has been used. (b) The scale has been changed to reveal details of the structure of the network that, in this case, is much more homogeneous and quite different from that observed in the nonconservative regime.

bolic networks, $P(k)$ has a power-law tail, $P(k)\sim k^{-\gamma}$. Such networks are called scale-free [11], and have called the attention of many researchers in the last years.

We simulated the OFC model in a square lattice, building graphs with a procedure very similar to what has been employed by Abe and Suzuki. Each site that gives birth to a new avalanche, of any size, is an epicenter; each epicenter defines a node, and every node is then connected to the node where the next epicenter occurs, establishing a link or edge between them. If two subsequent earthquakes start in the same cell (which does not happen if each site is a cell, but do happen with larger cells), we have a loop. After many avalanches this procedure generates a complex network (or graph), and we have studied some of its statistical properties.

After eliminating a transient of at least 10^6 events, we calculated numerically the distribution function $P(k)$ for the graph constructed from the time sequence of epicenters in the OFC model for different values of α and different lattice sizes. As the first and last sites are the only ones with an odd number of edges (if they do not coincide), they are eliminated. Our results for the distribution $P(k)$ can be seen in Fig. 1. It is clear that, if $\alpha<0.25$ [Fig. 1(a)], the distribution is scale-free for some decades, with an exponent γ that varies linearly with α (see Fig. 2), at least for values of α not too far from the conservative regime. In Fig. 1 some curves have been shifted upwards for the sake of clarity. They would coincide, except for a small change in the slope, otherwise. We have also observed that the network grows toward the inside of the lattice, with the most connected sites in the borders and the most inner sites being the last ones being added to it [see Fig. 3(a)]. The complex structure, however, is not a boundary effect. If we take out the border sites and adjust the scale, we see that the same spatial structure is reproduced [Fig. 3(b)]. Because one needs a growing network to observe the scale-free behavior [11], after a certain number of events, as a consequence of the finite size of the lattice, most of the sites of the lattice have already become part of the network. At this point the scale-free behavior starts to break.

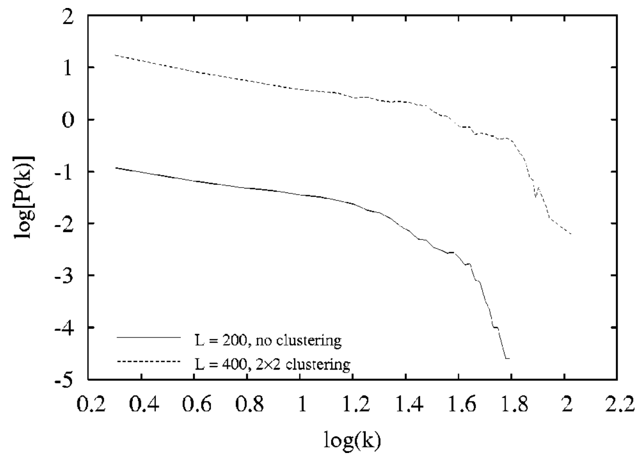

FIG. 5. Degree distribution $P(k)$, for $\alpha=0.249$, $L=200$, and 10^5 events, for different cell sizes. All log scales are base 10.(a) Continuous line: $L=200$ and each site of the lattice defines a cell. (b) Dashed line: $L=400$ and each four adjacent sites are in the same cell. The curve has been shifted upwards in the y axis for clarity.

If the system is conservative, however, the distribution function $P(k)$ has a well-defined peak, indicating a higher degree of homogeneity among the nodes [Fig. 1(b)]. Figures 4(a) and 4(b), which should be compared with Fig. 3, show the spatial distribution of connectivities (degree of the nodes) in the lattice. As expected, it is much more homogeneous. This homogeneous behavior is not destroyed if we change the statistic of events.

Finally, our findings also seem to be robust with respect to the cell size and the size of the lattice. If we increase the size of the cell, defining, for instance, four adjacent sites of the lattice as a unique cell, there is no change in the results, not even in the exponent γ that characterizes the degree distribution $P(k)$, as shown in Fig. 5. Also, there is no change if we increase the size of the lattice.

In conclusion, we have shown that the nonconservative version of the Olami-Feder-Christensen model is able to reproduce the scale-free network associated with the dynamics of epicenters observed on real earthquake data. The conservative version of the model displays a qualitatively different behavior, being closer to a random graph. The smallest degree of nonconservation seems to be enough to change the behavior of the model, since for $\alpha = 0.249$ we see that $P(k)$ has already a well-defined power-law behavior for some decades. Those findings, besides giving an indication of the robustness of this model to reproduce the dynamics of earthquakes, reproducing the experimental findings of Abe and Suzuki, present a completely different dynamical mechanism to generate a scale-free network. There is no explicit rule of preferential attachment, and the preferential attachment observed in the network is a signature of the model dynamics. Maybe the complete study of the properties of this network can help solve some still controversial aspects of the Olami-Feder-Christensen model and of self-organized critical behavior.

[1] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. A **38**, 364 (1988).
[2] H. Jensen, *Self-Organized Criticality* (Cambridge University Press, New York, 1998).
[3] D.L. Turcotte, Rep. Prog. Phys. **62**, 1377 (1999).
[4] Z. Olami, H.J.S. Feder, and K. Christensen, Phys. Rev. Lett. **68**, 1244 (1992).
[5] R. Burridge and L. Knopoff, Bull. Seismol. Soc. Am. **57**, 341 (1967).
[6] J.X. de Carvalho and C.P.C. Prado, Phys. Rev. Lett. **84**, 4006 (2000).
[7] K. Christensen, D. Hamon, H.J. Jensen, and S. Lise, Phys. Rev. Lett. **87**, 039801 (2001); J.X. de Carvalho and C.P.C. Prado, *ibid.* **87**, 039802 (2001).
[8] S. Lise and M. Paczuski, Phys. Rev. E **63**, 036111 (2001).
[9] G. Miller and C.J. Boulter, Phys. Rev. E **66**, 016123 (2002).
[10] S. Abe and N. Suzuki, Europhys. Lett. (to be published); e-print cond-mat/0308208.
[11] A.-L. Barabási and R. Albert, Science **286**, 506 (1999); R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002).
[12] D.J. Watts and S.H. Strogatz, Nature (London) **393**, 440 (1998).
[13] S. Hergarten and H.J. Neugebauer, Phys. Rev. Lett. **88**, 238501 (2002).
[14] J.X. de Carvalho and C.P.C. Prado, Physica A **321**, 519 (2003).
[15] G. Miller and C.J. Boulter, Phys. Rev. E **67**, 046114 (2003).